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The Birthday of the Old Man of Jiang County and Other Puzzles: Work in Progress on Liu Xin's *Canon of the Ages*

曰. 臣小人也. 不知紀年.
Zuozhuan (Xiang 30), where the cited words
are put into the mouth of the Old Man himself.

INTRODUCTION

There are parts of some books that one takes a long time to understand. Sometimes it is possible to convince oneself for a while that such material is not really important, that it ought not to delay one in trying to grasp the significance of the more accessible parts of the book. But it is hard to dismiss permanently the nagging suspicion that it *may* be important, that it *may* contain evidence that, if properly understood, would be crucial to one's understanding of the wider context in which it is embedded. So one keeps on coming back to it, making a fresh attempt, and retiring again in discouragement. And since for some reason it is not common for scholars to advertise their *ignorantia*, one may begin to suspect that one is alone in lacking in the mental equipment needed to do the job.

The writing that Liu Xin 劉歆 (c. 50 BC to 23 AD) called the *Canon of the Ages* (*Shi jing* 世經) has been one of those texts for me. Today it is found in *Han shu*, where the ancient editors placed it immediately after the technical specification of Liu Xin's Triple Concordance astronomical system, *San tong li* 三統曆.[1] Apparently, this text is the "listing 譜" referred to in the preceding chapter, in the editorial passage that runs:

> In the time of Chengdi 成帝 (r. 33–7 BC), Liu Xiang 劉向 (79–8 BC) went through all six astronomical systems [known in his day], and

[1] See *Han shu* 漢書 21B, pp. 1011–24, in the standard punctuated edn. (Beijing: Zhonghua shuju, 1962). The common rendering of *li* 曆 as "calendar" is not adequate for its use in such a context as this. Following the lead of Nathan Sivin, I mostly translate it as "astronomical system," or just "system."

> set out where they were right and wrong, to make his "Discussion of the Five Eras" (*Wu ji lun* 五紀論). His son [Liu] Xin went into all the subtleties of such matters, and wrote his Triple Concordance system, together with his Listing for explaining the *Spring and Autumn Annals*. His methods of inference[2] keep close to the essentials, so we have set them out here.[3]

As the reader will shortly see, the *Canon* consists of a series of references to events dated to the regnal years of various rulers from high antiquity up to the early Eastern Han (this latter material, which is little more than a list of the lengths of reign periods, cannot of course be by Liu Xin himself, but must have been editorially added). For each entry there is a number of statements drawn from ancient sources about astronomical conditions in those years, together with the results of retrodictions by Liu Xin evidently based on the use of more than one astronomical system. Such working as is given is very sparse, certainly not enough to show how Liu Xin arrived at the conclusions he gives. Although not all the material comes from the Spring and Autumn period, there is enough material from that period to justify the statement of the editors quoted above.

Now this material is not virgin territory, at least so far as East Asian scholars are concerned.[4] The problem for a Western reader is, however, that works of Chinese and Japanese scholarship are often not helpful starting points, since they assume a background of knowledge that such a reader is unlikely to possess. I have found that there is no

2 This word is an attempt to provide an equivalent of *tui* 推 in the original. The root meaning of *tui* is of course "push." However it is also used metaphorically in situations one "pushes" the boundary of one's knowledge outwards from things near at hand by extrapolate to obtain information about some thing or event at a cognitive distance. Thus in an astronomical context, the word may for instance occur in a heading to a set of instructions for finding out when lunar eclipses occur – *tui yue shi* 推月蝕. But it seems unnatural in English to talk about "inferring lunar eclipses." "Calculating" will not do, since it is a paraphrase rather than a translation: one may do calculations in order to *tui*, but not every instance of *tui* involves calculations. If we are talking about things that are to occur in the future, then "predict" seems to fit both root meaning and usage quite well. But very often (as in this case) one finds people talking about *tui* with reference to finding what happened a long time ago, for which "prediction" will hardly do. In such cases, I have therefore preferred in the main part of this essay to use the coinage "retrodiction" as a description of what is being attempted.

3 *Han shu* 21B, p. 979.

4 The principal relevant work of Qing scholarship is that of Li Rui 李鋭 (1768–1817), who left us detailed commentaries on several ancient astronomical systems. The material relating to the *Shi jing* in his collected works *Li shi yi shu* 李氏遺書 is to be found in *Zhongguo kexue jishu dianji tonghui* 中國科學技術典籍通彙, "Astronomy" (Tianwen 天文) (Henan Educational Press, 1995), vol. 2, pp. 731–41. For an excellent example of modern East Asian scholarship see Nōda Chūryō 能田忠亮 and Yabuuchi Kiyoshi 薮内清, *Kansho ritsurekishi no kenkyū* 漢書律暦志の研究 (Kyoto: Zenkoku shobō, 1947), pp. 137–79.

substitute for a first-hand experience of struggling with the ancient texts until one begins to find questions of one's own, and to sketch provisional answers to them. At that point one may turn gratefully to the assistance of one's East Asian predecessors, whose questions and answers will now make sense. Now, as a result of some years of working my way through the technical records of Han astronomy, as well as the copious material recording contemporary discussions of its development, I finally feel I have some control of the central problems of Liu Xin's text, at least so far as mathematical astronomy is concerned. I therefore offer here a sample of some of the results that follow from this investigation, drawn from a full-length translation and analysis of the *Canon* that is now complete in draft.

This is no more than a partial report on work in progress, and it necessarily leaves many important matters to one side. But I hope it will be acceptable as a small tribute to the work of a scholar whose familiarity with the milieu in which Liu Xin lived, wrote and calculated is unmatched at this end of the world, and probably anywhere else.

THE PLACE AND PURPOSE OF THE *CANON*

What is the *Canon of the Ages* for? In the opinion of the *Han shu* editors, Liu Xin wrote "in order to explain the Spring and Autumn [Annals] 以說春秋." Although Liu Xin does not give us any alternative explanation himself, I think this picture of his motives is at least inadequate, if not plain wrong. It does however bear some resemblance to the way in which his work has been treated in recent years in the context of discussions of early Chinese chronology, though the chronology has mainly been that of the early Zhou rather than of the Spring and Autumn period. In such writing, Liu Xin has been discussed as if his object had been to use astronomical calculation in order to establish a chronology of China up to his own day, using records of astronomical phenomena in trusted ancient texts as a means of fixing dates for the years in which those events occurred. In other words, his *Canon* would have been an early version of what is commonly known in the West as the Three Dynasties Project (*Xia Shang Zhou duandai gongcheng* 夏商周斷代工程), which over the last decade has worked to establish a firm chronological framework for the earliest periods of Chinese history.[5]

[5] A summary of the project's main results so far is given in the short, anonymous work *Xia Shang Zhou duandai gongcheng* 1996–2000 *chengguo baogao: jianben* 夏商周斷代工程 1996–2000 年阶段成果报告 (簡本) (Beijing: 2000).

The *Canon* has, however, been seen as a gallant failure rather than an example to be followed, for reasons shown most obviously in the fact that Liu Xin's date for the Zhou conquest of Shang was 1122 BC, whereas most modern estimates have been closer to the date of 1046 BC favored by the Three Dynasties project.[6]

My view is more or less the reverse of this: I believe that Liu Xin's aim in writing the *Canon* was to display the success of his Triple Concordance system in retrodicting known astronomical circumstances in historical periods for which he was able to establish a chronology based on non-astronomical considerations, such as a count of regnal years of successive rulers. That at least is how I believe he intended the *Canon* to be read – but by taking this view I do not intend to exclude two further possibilities:

1. That he designed his astronomical system precisely in order that it would retrodict ancient data correctly; this is in my view not only possible, but certain.
2. That where a choice was open to him, he made decisions on ancient chronology and on the use of ancient astronomical data in ways that made it easier to find a fit with his system.

Some evidence for both of these possibilities will emerge in the course of the discussions of sample material given below.

THE SYSTEMS USED

Liu Xin uses two main astronomical systems in the *Canon*, while referring to at least two others. His main reference resource is his own creation, the Triple Concordance system (*San tong li* 三統曆), although it is mentioned by name only twice. The other system is one that he calls the Yin system (*Yîn li* 殷曆), which is named relatively frequently, presumably because it is referred to explicitly for comparison, and is not the system he wishes to be taken as his basic reference.

Triple Concordance system

The Triple Concordance astronomical system was created by Liu Xin on the basis of the earlier "Grand Inception astronomical system" (*Taichu li* 太初曆) that had been in official use from 104 BC. It retained the basic constants that determined months and years in its predecessor,

6 A very fair and properly generous estimate of Liu Xin's work seen in this light will be found in Jiang Xiaoyuan 江曉原 and Niu Weixing 纽衛星, *Huitian: Wu Wang fa Zhou yu tianwen lishi niandai xue* 回天武王伐紂與天文歷史年代學 (Shanghai: People's Publishing House, 2000), pp. 147–83.

but added material for the prediction of planetary motions Thus it was necessary to use a system origin much earlier than the date of 104 BC used by the Grand Inception system, in order to find an instant when all elements of the system were at their starting points.[7]

Yin System

There is no full independent description in ancient sources of the system that Liu Xin calls the Yin system (*Yin li* 殷曆). It is however possible to recognize it as a member of the family of systems known generically as *Si fen* 四分 "Quarter [day]" systems, a name apparently based on the fact that all of them postulate basic figures that imply that winter solstices recur at intervals of precisely 365¼ days. Such systems all share a common set of constants, and apart from certain minor details their calculating procedures are apparently the same. The only significant difference between systems lies in the starting conditions from which calculations begin. From explicit statements made in Liu Xin's *Canon* it is possible to reconstruct these for the Yin system quite easily. As a result the whole of the system can be reconstructed by simply changing the initial conditions for the one "Quarter [day]" system for which a full description has been preserved, which is the "Han system" (*Han li* 漢曆).[8]

Other Systems Mentioned

As will be seen below, at one point Liu Xin refers briefly to what appears to have been the system of the "Quarter [day]" type prepared by Sima Qian 司馬遷 in connection with the astronomical reforms of 104 BC, but not officially adopted. In another passage, not discussed here, he criticizes a system that while calling itself a *Yin li* differs from that used by him elsewhere.

CALCULATION AND CALCULATING AIDS

The Triple Concordance and Yin systems, like all other ancient Chinese astronomical systems, have three essential elements:

[7] A full description of the Triple Concordance system is to be found in *HS* 21B, pp. 991–1011, a book which was largely complete on the death of its main author and editor, Ban Gu 班固 in 92 AD.

[8] This was in official use from 85 AD. The technical specification is set out in the third *zhi* 志 "monograph" of *Hou Han shu*, the monographs of which, as we have them today, are in fact borrowed from an earlier history of the Eastern Han dynasty, now lost, *Xu Han shu* 續漢書 of Sima Biao 司馬彪 (ca. 240–ca. 306 AD). See *Hou Han shu* 後漢書 (Beijing: Zhonghua, 1963), *zhi* 3, pp. 3055–82.

1. Each has a defined moment of "System Origin" *li yuan* 曆元, or epoch, at which all elements in the system are postulated to have been at certain simple initial states, typically including conjunction of sun and moon at the instant of winter solstice, this coinciding with midnight beginning the first day of the first month of the year according to the Celestial month-count (on which see below). It is implied, though not usually stated, that a lunar eclipse occurs at the first full moon of the system following the moment of origin, which is of course a new moon. Sometimes the system origin is referred to as the "High Origin" (*shang yuan* 上元) to distinguish it from less distant moments at which some but not all of the initial conditions of a system may repeat. In some but not all cases the system origin is so far in the remote past that it is obvious that its only purpose is to enable one to make predictions in the present day, and that it cannot have been based on any record of conditions from the epochal date itself.
2. A set of constants defining the ways in which the entities whose behavior is to be predicted move over time. The most important constants of a system are given in a listing at the start of the text specifying that system. There may also be tables giving data for more complex parts of the system. Despite the constant temptation to recast ancient mathematics into modern forms, it is important to note that in the Han the constants given do not refer explicitly to the time intervals that nowadays we would call the mean synodic month or the tropical year.[9] Although implied values for such quantities can be deduced from the constants given, they are not the actual objects of attention or manipulation.
3. A set of arithmetical procedures for using 1 and 2 to predict the state of all elements in the system at any instant other than the system origin.

The object for which these systems are designed is not explicitly astronomical so much as administrative. For any given year, the basic output of the system consists of the following:

[9] The synodic month (which is not a whole number of days) is the interval between two successive conjunctions of sun and moon ("new moons"). The complicated behavior of the sun-earth-moon system means that this interval is not a constant, but varies by about a quarter of a day on either side of an average value of 29.5306 days, the "mean synodic month." All calendar months must contain a whole number of days, and so to keep in step with the sun and moon calendar months will mostly alternate between 29 and 30 days, with occasional pairs of 30-day months in succession. In the period we are discussing, astronomical systems did not attempt to follow the real instant of conjunction (the "true conjunction"), but simply predicted a series of equally spaced conjunctions which followed the true conjunctions closely enough for practical purposes ("mean conjunctions"). The tropical year is the interval between successive passages of the sun past the spring equinox, which is 365.2422 days; ancient Chinese practice used the winter solstice as the preferred mark-point, but the relevant interval is of course the same.

a. The sexagenary cyclical names of the days on which the successive months of the year begin. Each day of the month begins at midnight, and ideally the first day of each month should contain somewhere within itself the instant at which sun and moon are in conjunction. Such months are therefore known as "lunar months" (in distinction from the purely conventional months of the Gregorian calendar), although since they contain whole numbers of days their lengths are only on average equal to the mean synodic month, and will be either 29 or 30 days. In the period we are discussing, the effective assumption that such conjunctions recur at equal time intervals implies we are dealing with the mean rather than true conjunction.
b. The number of months in the year, usually 12, but 13 when an extra "intercalary" month has to be inserted to keep the year used to date events in the human world in step with the cycle of the seasons.
c. A subsidiary aim might be to find the days on which the 24 *qi* 氣 fall. In the context of astronomical calculation, these are the 24 instants, beginning with the instant of winter solstice, that divide the interval between one winter solstice and the next into 24 equal parts. This interval, the tropical year, is the period at which the seasons recur, and is about 11 days longer than twelve lunar months. Thus a series of twelve-month years will get further and further in advance of the seasons. After three years the "advance" will be slightly greater than one month. Throughout the *Canon*, it is usually only the first *qi*, winter solstice, that is mentioned. It is worth noting however, that the odd-numbered *qi* , with winter solstice as number 1, have a special significance. These are the "Medial *Qi*" (*zhong qi* 中氣); for Liu Xin if each successive month of the year has a Medial Qi in it, all is well. But if the progressive slippage between lunar months and Medial Qi reaches the point where one month has no Medial Qi in it, the under the Triple Concordance System that month is counted as an "intercalary month" (*run yue* 閏月) with the same number as the preceding month to allow the two sequences to get back into step. This will automatically bring the months back into step with the seasons. One of Liu Xin's complaints about the early records he uses is that intercalation has not been systematically kept up, so that the numbers of the ancient months do not correspond exactly to his calculations.[10]

[10] In the earliest periods for which we have evidence it seems that intercalary months were inserted more or less whenever it was noticed that the calendar and the seasons were out of step. But an analysis of the placing of intercalary months suggests that this was beginning to be done more systematically from the early 6th-c BC. See Yabuuchi Kiyoshi 薮内清, *Chūgoku no tenmon rekihō* 中国の天文暦法 (Tokyo: Heimarusha, 1969), pp. 278–80. For Liu Xin, of course, a "missed" intercalation is one that has not been done when his Triple Concordance system would have required it.

I have prepared for publication elsewhere a full, annotated translation of the main content of the ancient texts describing the first three Chinese astronomical systems known to us in full detail – the Triple Concordance and Han systems already mentioned, together with the Uranic Manifestation system of Liu Hong 劉洪 (c. 135–210 AD). While that publication is awaited, much useful information on the workings of such systems may be found in Nathan Sivin's short monograph *Cosmos and Computation in Early Chinese Mathematical Astronomy.*[11] Here I shall simply describe the basic form of a typical calculation to produce data for a given year.

The first, and indispensable, step is to know how many years have elapsed from the moment of system origin until the beginning of the "target year" for which we seek to predict calendrical data. If this is a large number, we may simplify our calculations by casting out numbers of years over which the initial conditions of the system are known to repeat, in their entirety or at least in so far as they concern us. That will keep the numbers in our calculations as small as possible, thus increasing the convenience and speed of hand calculation and reducing error.

Thus for instance, in the Triple Concordance system, in the interval known as an Origin [cycle] (*yuan* 元), all basic solar and lunar conditions of the system repeat: we come back once more to a year in which winter solstice and conjunction of sun and moon coincide with midnight beginning a day whose sexagenary name is *jiazi* 甲子, number 1 of the cycle, and which begins the first month of the year.[12] The number of years in this period is given by the Origin Factor *yuanfa* 元法, 4,617. So we may cast out (subtract) whole multiples of 4,617 from the year count to begin with. Next we may exploit the fact that each Origin contains three Concordances (*tong* 統) of 1,539 years each, at the start of which (the Concordance Heads [*tongshou* 統首]) epochal conditions all repeat except for the sexagenary number of the day, which shifts onwards by 20 each concordance to bring us back to the start after three Concordances have passed and we are at the start of a new Origin. Thus if the first Concordance of an Origin begins on day *jiazi.*1, the second will begin on day *jiashen.*21, the third on day

[11] Published in Leiden, 1969; rpt. with retrospect in Sivin, *Science in Ancient China* (Variorum, 1995), chap. 2.

[12] Mathematically speaking, one might simply replace all "sexagenary day names" such as *jiazi* or *guihai* by the appropriate numbers – which would be 1 or 60 respectively in this case. But since I find it impossible to convert rapidly from names to number and back again in my head, I find that where calculations are involved it facilitates reference between translation and original if one gives sexagenary days as *jiazi.*1, *guihai.*60 and so on.

jiachen.41, and the next (the first Concordance of the next Origin) on day *jiazi*.1 once more.[13]

Suppose, for example, that we are at the start of a year which is numbered 20,582 in a sequence in which the year commencing at High Origin was number 1. Thus there have been 20,581 completed years so far. Now:

$$20{,}581 = 4 \times 4617 + 2113$$

So we are now 2,113 years from the start of the current Origin. Further,

$$2{,}113 = 1539 + 574$$

So one full Concordance has been completed, and we are 614 years from the start of the second Concordance in the present Origin.

Having cast out any full Concordances, what we have is the number of years from the start of the current Concordance up to the day on which the target year begins. We now proceed to turn this interval into months, and then into days. The conversion into months is easily done, using a shorter cycle of repetition, the Rule, of length 19 years or 235 months, after which winter solstice falls again on the first day of the first month of the year. Since

$$235 = 12 \times 19 + 7$$

it is clear that we should expect 7 intercalary months in the course of 19 years.

Thus if we take the number of years so far, and use the whole number part of the result of multiplying by 235 and dividing by 19 (ignoring any remainder), we shall get the number of months up to the start of the target year. To convert to days, we use the fact that according to the Triple Concordance, after 81 months or 2,392 days conjunction falls again at midnight beginning a month. Thus we multiply "accumulated months" just calculated by 2,392, and take the whole number part of the result of dividing by 81 to get the days since the start of the last Concordance. If we cast out multiples of 60 from this, the result will tell us how many days we have to move on from the sexagenary day number of the start of the Concordance to find the sexagenary day number on which the first day of the first month of the target year falls. Other calendrical data can be generated using similar procedures.

Using our example, we may find:

$$574 \times 235/19 = 7{,}099$$

[13] The monograph by Sivin referred to earlier illustrates these points using a graphical portrayal, which some readers may find helpful. The discussion given here sticks as closely as possible to the traditional Chinese mode, which is more arithmetical in style.

ignoring any remainder. Thus there have been 7,099 months since the start of the Concordance. Further:

$$7099 \times 2392/81 = 209{,}639$$

ignoring any remainder. Thus there have been 209,639 days since the start of the current Concordance. Further:

$$209{,}639 = 3493 \times 60 + 59$$

Thus the first day of the year we are considering will have a sexagenary day number 37 further on than the day with which the Concordance began. The current Concordance is the third one of the current Origin, and so it began on day *jiachen*.41. Thus the current year begins on a day whose number is given by *guimao*.40.

In systems of the Quarter [day] type, the principles are the same, but the lengths of the basic cycles are different. The Origin [cycle] to be cast out first from the year count is 4,560 years; unlike the Triple Concordance Origin, this is a multiple of 60 and so gives repetition of the sexagenary number of the year as well as of the other usual conditions. The next level down is the Era (*ji* 紀) of 1,520 years, of which there are 3 to each Origin. At an Era Head, all conditions of the System Origin repeat apart from the sexagenary number of the year. Having cast out Eras, we have the years since the last Era Head. We could if we wished then convert to months and days by similar procedures to those in the Triple Concordance, but in fact Quarter [day] systems simplify calculations by using a smaller interval, the Obscuration (*bu* 蔀), which is 76 years, and gives repetition of the coincidence of conjunction and winter solstice at midnight on the first day of the first month. Then we convert to months, using the same Rule of 235 months or 19 years as before, and to days using the fact (prescribed by the system) that an Obscuration in a Quarter [day] system contains exactly 27,759 days in 940 months. As before, we cast out full 60-day cycles from the result, to get the shift in cyclical day number for the first day of the target year compared with those at the start of the Obscuration.

The difference in the figures used in these two systems may obscure the fact that the results they produce are very similar. To take one example only, in the Triple Concordance the average length of a month is clearly given by:

$$2392/81 \text{ days} = 29\,43/81 \text{ days} = 29.531 \text{ days to five significant figures,}$$

while in the Quarter [day] systems the length is:

$$27759/940 \text{ days} = 29\,499/940 \text{ days} = 29.531 \text{ days to five significant figures.}$$

Clearly therefore the two systems agree in this respect to within about 1/1000 day, or about one and a half minutes. It is only in the long term that noticeable discrepancies will appear. A modern value for the mean synodic month is identical, to the same precision.

But before we can begin our calculations for any given year, we do need to know where (or rather when) the system origin we are using actually is. Fortunately Liu Xin makes this quite plain. In what is probably the final section of the extant *Canon* to have been written by him as opposed to the *Han shu* editors who adapted his text, he follows his story into the reigns of the Western Han emperors. For the first year of the Han (which corresponds quite closely to the year 206 BC of the Julian calendar, although it began late in 207 BC) he says:

> This was distant from the High Origin by 143,025 years. 距上元年十四萬三千二十五歲.[14]

Thus the first year of the Triple Concordance system is:

(206+143025) BC = 143231 BC

As will be explained shortly, this means the actual starting point of the system was the winter solstice of 143,232 BC immediately preceding this Julian year.[15] Of course this is many Origins before Liu Xin's present. If we subtract whole Origin Factors (4,617 years), we find that the most recent Origin Head was at the winter solstice preceding 104 BC. This is not surprising, since it is well known that the core of the Triple Concordance system was the Grand Inception system inaugurated in 104 BC that did indeed have its origin at that point. Liu Xin's much more distant High Origin is needed in order for him to be able to incorporate predictions of planetary motions.

Later, in an entry for a year corresponding to late 48 BC to 47 BC, he says:

> In the second year of the Chuyuan reign period of Yuandi, the 11th month, [day] *guihai*.60 was conjunction and winter solstice [according to the Triple Concordance system]. The Yin system made this *jiazi*.1, which was taken as an Era Head. 元帝初元二年十一月癸亥朔旦冬至, 殷曆以爲甲子, 以爲紀首.[16]

[14] *HS* 21A, p. 1023.

[15] The calendar nowadays in international use follows the model laid down in the reforms sponsored by Pope Gregory in 1582. For dates before that, it is conventional to use the Julian Calendar introduced by Julius Caesar in 45 BC, even for dates well before the Julian reform (the so-called "proleptic" Julian calendar).

[16] *HS* 21A, p. 1024.

So if 47 BC is a year beginning with an Era Head, for the Yin system we can see that the preceding Era Head was in:

(47+1520) BC = 1567 BC

Since the only difference between one Era Head and another in Quarter [day] systems is the sexagenary number of the year, then we can treat this as a system origin for all the relatively simple processes needed in the *Canon*. If we really are interested in knowing where an Origin begins, all we have to do to see if 1567 BC is an Origin Head is to check the year number. Since this turns out to be *jiayin*.51 in 1567 BC, and a well informed source in the Eastern Han tells us that the Yin system had its origin in such a year,[17] we may take this as an Origin Head.

Further evidence that this was indeed the Yin system as recognized by Western Han scholars comes from the record of a controversy that took place in 78 BC, when objections were raised to the Grand Inception system by someone who claimed to possess an astronomical system stemming from the Yellow Emperor. This system was claimed by the astronomical officials to be in fact identical to the "Yin system" in their own official files. Now that system apparently ran ¼ day behind the Grand Inception system – which is exactly how the Yin system did indeed place the winter solstice and conjunction of 105 BC in relation to the Grand Inception epoch.[18] Indeed, given that the 78 BC controversy was only settled after three years of work by the astronomical officials, it is perhaps easier to see why Liu Xin would have thought it proper to include systematic comparison data from the Yin system in his *Canon*. Is there also perhaps a possibility that the entire enterprise of the *Canon* may in fact be based on the work of his colleagues several decades earlier? There is just no way of telling.

THE JUPITER STATIONS

The account given so far describes the way that the core of the two astronomical systems mainly used in the *Canon* calculate days, months, and years. The Triple Concordance system does however have one other important feature, which is its ability to calculate the position of Jupiter (*sui xing* 歲星, or "Year Star") with reference to its twelve "stations" (*ci* 次). These are defined as twelve divisions, of either 30 or 31 *du* in length, of the circuit of the 28 lodges (*xiu* 宿). The table, below,

[17] *Hou Han shu*, *zhi* 2, p. 3038.

[18] See *HS* 21A, p. 978, and also the discussion in Christopher Cullen, *Astronomy and Mathematics in Ancient China: The Zhou bi suan jing* (Cambridge: Cambridge U.P., 1996), p. 30.

sets them out; its data are taken more or less directly from the listing of the Jupiter stations in the *Han shu* description of the Triple Concordance system. The rule for locating the station in which Jupiter is to be found in a given year is a simple one:[19]

> To predict the location of the Year [Star], set out [years] since the High Origin, excluding the year sought (so that, for example, if the year of interest is the 1,523d year counting the year beginning at High Origin as number 1, set out 1,522). Cast out what fills Year Number [for Jupiter] (1,728) (that is, cast out whole multiples of 1,728). Multiply what does not fill [the Year Number] by 145 (that is, multiply the remainder by 145). Take 144 as the factor. Obtain 1 for each accord with the factor (that is, find how many times 144 goes into the previous result). 推歲所在, 置上元以來, 外所求年, 盈歲數, 除去之, 不盈者以百四十五乘之, 以百四十四爲法, 如法得一.
>
> That is called Accumulated Stations. What does not fill [the factor] is called Stations Remainder. If Accumulated Stations fills 12, cast it out (since the cycle of stations is twelve stations long). 名曰積次. 不盈者名曰次餘 . 積次盈十二, 除去之.
>
> What does not fill is called Corrected Stations. Count off this number from *xingji* [station number 1], and outside the exhausted count, then that is the station where it is (so that, for example, if Corrected Stations is 7, meaning we have moved 7 stations from the starting point, we start with *xingji* as number 1, and "exhaust the count" at the 7th station, Chunshou. Jupiter will be found in the 8th station, Chunhuo). 不盈者名曰定次. 數從星紀起, 算盡之外, 則所在次也.

The essence of this is clearly that in 144 years Jupiter moves through 145 stations, starting at High Origin from the beginning of the first station, Xingji. (Note that this point is 15 *du* to the west of the winter solstice, which is where the sun is at High Origin. For Liu Xin, a planet just becomes visible when it reaches 15 *du* from the sun.) The Year Number 1728 is 12x144, so in 1,728 years Jupiter moves through 12x145 stations. Since this is a multiple of the 12 stations, we may cast it out without affecting the result, which is why we are told to start by casting out Year Numbers.

[19] *HS* 21B, pp. 1005–6. I give an extremely literal translation below, in which the emphasis is on showing how the Chinese formulates instructions for carrying out the necessary calculations. The result is not elegant or natural English, but then the same could be said of the original Chinese, which would probably not have been readily understood by someone unfamiliar with such calculations. I have interspersed a few comments to help make things clear.

The Jupiter Stations

The translations of lodge and station names are those proposed in Edward H. Schafer, Pacing the Void: T'ang Approaches to the Stars *(Berkeley: 1977), table 2, p. 75.*

	JUPITER STATION	START OF STATION COUNTED FROM W. SOLSTICE, IN *DU*	LODGE IN WHICH STATION STARTS	*DU* OF LODGE FIRST IN STATION	WIDTH, IN *DU*
1	Xingji 星紀 "Star Chronicle"	−15	Dou 斗 "Dipper"	12	30
2	Xuanxiao 玄枵 "Murky Hollow"	15	Nu 女 "Woman"	8	30
3	Quzi 諏訾 "Loggerhead Turtle"	45	Wei 危 "Rooftop"	16	31
4	Jianglou 降婁 "Descending Harvester"	76	Kui 奎 "Straddler"	5	30
5	Daliang 大梁 "Great Plank-bridge"	106	Wei 胃 "Stomach"	7	30
6	Shichen 實沉 [name of deity in Orion]	136	Bi 畢 "Net"	12	31
7	Chunshou 鶉首 "Quail Head"	167	Jing 井 "Well"	16	30
8	Chunhuo 鶉火 "Quail Fire"	197	Liu 柳 "Willow"	9	31
9	Chunwei 鶉尾 "Quail Tail"	228	Zhang 張 "Spread"	18	30
10	Shouxing 壽星 "Longevity Star"	258	Zhen 軫 "Axletree"	12	31
11	Dahuo 大火 "Great Fire"	289	Di 氐 "Base"	5	31
12	Ximu 析木 "Split Wood"	320	Wei 尾 "Tail"	11	30
[1]	[Xingji "Star Chronicle"]	[350]	[Dou "Dipper"]	[12]	[30]
				TOTAL	365

Since references to the station of Jupiter in a given year are common in the ancient sources used by Liu Xin, such calculations are clearly important to him, and he makes them frequently. One important point must be mentioned, although it need not detain us for long here: the Jupiter station records in the *Zuo Zhuan* are clearly not records of contemporary observations of Jupiter in the years to which they are dated, since they are so far from the actual positions of Jupiter yielded by accurate modern calculation as to make this impossible. It was at one time suggested that the close correspondence between these records and Liu Xin's calculations proved that he had forged the records and interpolated them into the text. Nowadays it seems more likely that this was done some centuries before Liu Xin's time by someone who extrapolated backwards from his own observations on the basis of some simple rule such as that Jupiter moved one station a year. When Liu Xin worked on this material, he presumably constructed his system so that it fitted what he took to be reliable ancient data, and so the result was his rule given above, which is, as it happens, not a very accurate rep-

resentation of Jupiter's behavior – in fact it would be better to say that it moves an extra station in the course of 84 years rather than 144.[20]

THE PROBLEM OF THE NEW YEAR

One further matter, already hinted at, remains to be settled before we can turn to the text of the *Canon* – when does a new year begin? Since a year in the civil dating sense contains a whole number of months, it must clearly begin at the midnight that starts the first day of some particular month. But which month is to be taken as the literally the "standard month" (*zheng yue* 正月), or (paraphrasing, as hereafter) "first month"? Confusingly, there are in fact at least three possible months that could be a *zheng yue*, and what is more there are three different ways of referring to each of them. There is however one fixed point in all this, and that is clearly shown at the moment of System Origin. The month that begins then is the first one for astronomical purposes, and of course it begins with the instant of winter solstice. This is called the *tian zheng* 天正 "Celestial first [month]," as are all months that contain the winter solstice in other years, although they will only rarely begin with it. For the period discussed in most of the *Canon*, and indeed up to the present day so far as the traditional Chinese New Year is concerned, if the Celestial first month is number one, it is however month number three that begins the year for purposes of civil dating. It is this month that I customarily call the Civil First Month, and from which the months of the civil year are numbered. I list here the first three months of the "Celestial" month-count, with the three different labels that may be attached to them:

	COSMIC	DYNASTIC	DIPPER ESTABLISHMENT *dou jian* 斗建
1	天正 celestial	周正 Zhou	*zi* 子
2	地正 terrestrial	商正 Shang	*chou* 丑
3	人正 anthropic	夏正 Xia	*yin* 寅

These are the months conventionally called the *san zheng* 三正 "3 first [months]." What I have called here the "cosmic" system uses the names of the triad of powers made up of Heaven, Earth, and Man. The "dynastic" system uses the names of the three great dynasties of high antiquity, of which the oldest, the Xia, is named last and the most recent, the Zhou, is named first. Those who used this system conven-

[20] For further discussion, see Jiang and Niu, *Huitian*, pp. 156–59.

tionally supposed that each of these three dynasties had begun its civil year in the month bearing its name. The third system is that of the *dou jian* 斗建 "dipper establishment." In reality it is simply a system of labeling all the twelve months of a year without an intercalation using the set of 12 cyclical signs listed earlier, beginning with the first sign *zi* 子 for the month containing the winter solstice. The connection with the Dipper comes from a conventional explanation repeated by commentators down the centuries, according to which the handle of the Northern Dipper seen at dusk in each of the twelve months of a year without an intercalary month shifts points successively through the twelve directions designated by the twelve cyclical signs, with *zi* corresponding to due north, *mao* 卯 to due east and so on. Of course the Dipper does go through an annual cycle of this general kind, but the varying time of dusk throughout the year, combined with the shifts of the months relative to the solar seasons (to say nothing of the secular shifts caused by precession) destroy any regularity of the sort usually claimed. The lack of realism in these schemes is made clear when some commentators claim that in an intercalary month the Dipper points "between two cyclical signs."

Though the point is not important for the *Canon* it must be mentioned that under the Qin, and in the Han up to 104 BC, the civil year began in a month that was not one of the three above, but which was the month before the Celestial first month, the tenth month of the Xia count. Despite this, the months retained their Xia numbering for dating purposes. The result, a year beginning in its own tenth month, is a fruitful source of confusion for those beginning to study ancient calendrical material.

WORKING THROUGH THE CANON

For an ancient astronomer such as Liu Xin, there was no alternative to carrying out the calculations specified above through step-by-step manipulations of counting rods, possibly laid out on a conveniently marked counting board. One slip at an early stage, and everything that followed would be thrown out of joint. While an expert and disciplined hand calculator can compute rapidly and accurately by these means, modern scholars (at least this modern scholar) have neither the expertise and training nor the time to do this kind of thing. As such a modern, I have turned to modern technology to help me out, and have constructed electronic spreadsheets that automatically carry out all the steps prescribed by the a given astronomical system to find all the calendrical

data for any target year, once the target year has been typed into the appropriate cell of the spreadsheet.[21] As a result one can easily try out several different systems and several different years in quick succession, with the assurance that once the system has been constructed, tested and debugged it will give accurate results ever after.[22]

The Beginning of the Count

The *Canon* begins with some passages of genealogy without any numerical data, carrying us down from the time of the Yellow Emperor. When we reach the reigns of Yao, Shun and Yu the Great, we have reign-lengths for each of them. Finally we are told of the Xia dynasty founded by Yu:

> It passed down by inheritance through 17 kings, and endured 430 years. 繼世十七王, 四百三十歲.[23]

Shortly after this, the following sequence of material begins to deploy the astronomical systems available to Liu Xin:

> Tang the Successful: In the *Book of Documents*, the *Declaration of Tang* [recounts] how Tang attacked Jie of Xia. Metal produces water, so he acted in the virtue of water.[24] His realm was named as Shang, but later it was called Yin. From the Triple Concordance High Origin to the year when Jie was attacked is 141,480 years; the Year [star] was at Dahuo, in *du* 5 of Chamber.[25] 成湯: 書經湯誓, 湯伐夏桀. 金生水, 故爲水德. 天下號曰商, 後曰殷. 三統, 上元至伐桀之歲, 十四萬一千四百八十歲, 歲在大火房五度.

The High Origin of the Triple Concordance is the winter solstice late in 143232 BC, preceding the Civil Year whose first month according to the usual civil (Xia) count was in spring 143231 BC. Finally, 141,480 years from Civil Year 143231 BC takes us to Civil Year (143231–141480) BC = 1751 BC, which is therefore the year of the attack on Jie, when Tang came to power.

[21] I happen to be using Microsoft Excel on a Mac with OS 10.2.4, but almost any other spreadsheet application would work as well.

[22] I hope by the time this article appears in print that the reader should be able to download a copies of the relevant spreadsheets from the website of the Needham Research Institute at <www.nri.org.uk>. It should not therefore be necessary to take on trust the results I here ascribe to the use of spreadsheets for both the Triple Concordance and the Yin systems.

[23] *HS* 21A, p. 1013.

[24] I render *de* as "virtue" here in the sense that one speaks of "the evil virtue" (i.e. power or strength, *virtus*) of a poison. The five *de* of water, fire wood metal and earth are of course more commonly known as the five *xing* 行 "phases" or as some would prefer "processes."

[25] *HS* 21A, pp. 1013–14.

The Triple Concordance system spreadsheet confirms that the Jupiter station for this year (in fact at the immediately preceding winter solstice of 1752 BC) is Dahuo, with Jupiter in the fifth *du* of Chamber as stated. We may easily confirm this result manually: at the High Origin of the Triple Concordance system, Jupiter is at the start of station 1, Xingji 星紀, whose first *du* is the twelfth *du* of the lodge Dipper. Now in 144 years, Jupiter moves through 145 stations. So the number of stations through which Jupiter has moved since the High Origin at the winter solstice preceding the civil year 1751 BC is given by

$$141480 \times 145/144 = 142{,}462.5$$

and $142{,}462_{\text{mod}12} = 10$

This means that the solsticial position of Jupiter will be 10.5 stations from the start of station 1. So the Jupiter station is number 11, Dahuo, and we are half way through it. The width of this station is 31 *du*, and its first *du* is the fifth *du* of Base. So we count 15.5 *du* from the start of the station, which is at the start of the 5th *du* of Base, so that we arrive at (extended) Base 19.5 *du*; Base is 15 *du* wide, so we are 4.5 *du* from the start of the next lodge, Chamber, and hence in its 5th *du*, as stated.

> So the *Tradition* says: "Dahuo is the star of Ebo (a Shang ancestor of the same generation as Yao), and really [indicates] the reckoning (that is, fate) of the people of Shang." Later it acted for Tang the Successful, who when it came to him to pass away in his generation, had been acting as Son of Heaven for thirteen years. In the Shang twelfth month, day *yichou*.2 , conjunction was on the day of winter solstice. 故傳曰: "大火, 閼伯之星也, 實紀商人." 後爲成湯, 方即世崩沒之時, 爲天子用事十三年矣. 商十二月乙丑朔旦冬至.[26]

Tang attacked Jie in 1751 BC, according to the discussion above. Thus 1750 BC was his first full year of power, and his thirteenth year as Son of Heaven would be 1738 BC, since 1737 + 13 = 1750. (The reader may not experience the moments of confusion I sometimes feel about such simple calculations; such readers may simply skip my more labored passages. Others will understand.)

The Triple Concordance system does indeed predict day *yichou*.2 for the winter solstice of December 25, 1739 BC, which is the one preceding the Civil Year 1738 BC, and this is the first day of the first Celestial month, when the conjunction falls. In the conventional sequence recognized by Liu Xin (as distinct from historical reality), the Shang

[26] Ibid.

twelfth month is equivalent to the first Zhou month (which is the first Celestial month), or the eleventh Xia month, since the sequence of first months is Zhou–Shang–Xia. The fact that winter solstice and conjunction fall on the same day show that this is a Rule Head for the Triple Inception, but since the solstice and conjunction fall at different times of day it is not a Concordance.

> Thus the Preface to the Book of Documents says "When Tang the Successful had died, that was the first year of Taijia, who made Yi Yin make the Injunctions of Yi." The chapter Injunctions of Yi says "In the first year of Taijia, the twelfth month, day yichou.2, conjunction, Yi Yin did sacrifice to the former kings, and made rich offerings from his flocks[27] at the Square Numen." This says that even though this was done in service to Tang the Successful, Taiding, and Waibing he took the extraordinary measure at the winter solstice of sacrificing to the former kings at the Square Numen as the correlates of Shangdi. This is a year when conjunction falls at the winter solstice. Ninety-five years later, in the twelfth Shang month, day *jiashen*.21, conjunction fell at the winter solstice, and the Surplus Parts were extinguished: this was a Meng Concordance (that is, the third in the current Origin). 故書序曰: "成湯既沒, 太甲元年, 使伊尹作伊訓." 伊訓篇曰: "惟太甲元年十有二月乙丑朔, 伊尹祀于先王, 誕資有牧方明." 言雖有成湯、太丁、外丙之服, 以冬至越弗祀先王于方明」言雖有成湯、太丁、外丙之服, 以冬至越弗祀先王于方明以配上帝, 是朔旦冬至之歲也. 後九十五歲, 商十二月甲申朔旦冬至, 亡餘分, 是爲孟統.[28]

The first year of Taijia appears to have been counted as the same as the last year of Tang, Civil Year 1738 BC, since the same solsticial conditions are stated for both.

The "surplus parts" referred to here are the so-called "lesser surpluses," the fractional parts of a day by which some astronomical event such as the winter solstice or a conjunction falls after midnight of the day in question. When these parts are zero, the event falls at midnight. Ninety-five years from Civil Year 1738 BC takes us to Civil Year 1643 BC, for which indeed the Triple Concordance system predicts the conditions given here, with no Lesser Surpluses for *qi* or conjunction at the winter solstice, which precedes it. This is the Head of the third

[27] This is a highly conjectural rendering of the four characters 誕資有牧. Fortunately the exact nature or the rites conducted is not a crucial point in this context.

[28] Ibid., p. 1014.

Concordance (*meng tong* 孟統) of the current Origin of the Triple Concordance system.[29]

> From the attack on Jie to Wu Wang's attack on Tchou [romanised conventionally here to avoid confusion with the Zhou 周 dynasty] is 629 years, so the *Tradition* says of Yin "Its years were six hundred." 自伐桀至武王伐紂, 六百二十九歲, 故傳曰殷 "載祀六百."[30]

The attack on Jie took place in Civil Year 1751 BC, and (1751–629) BC = 1122 BC, which is indeed the year when Liu Xin places the attack on Tchou, as later specified.

Sima Qian's Quarter [Day] System

Shortly after the material discussed above, the following brief reference occurs:

> [According to the] Quarter [day system], from the High Origin to the attack on Jie is 132,113 years. The 88th Era, the *jiazi*.1 *fu* [sc. *bu* 蔀, Obscuration] Head,[31] enters 127 years after the attack on Jie. 四分，上元至伐桀十三萬二千一百一十三歲, 其八十八紀, 甲子府首, 入伐桀後百二十七歲.[32]

Which of the wide range of "Quarter [Day]" systems is referred to here? Now the attack on Jie is placed in 1751 BC. So what Liu Xin calls the "Quarter [day]" High Origin is in

> (1751+132113) BC = 133864 BC .

An Era is 1,520 years, and

> 87x1520 = 132,240,

so the 88th Era begins in (133864–132240) BC = 1624 BC, which is thus an "Era Head."

> 1751–1624 = 127, as stated.

If we set an epoch of 133864 BC, with epochal day *jiazi*.1 marked by coincidence of winter solstice and conjunction at midnight, as usual, then this year of 1624 BC is indeed a *jiazi*.1 Obscuration Head, with no Surpluses (Greater or Lesser) for conjunction or solstice at the Celestial New Year preceding it in late 1625 BC, since every Era begins with such

[29] The distance of 1643 BC from High Origin is 141,588 years. Now:
141588 = 30x4617 + 2x1539, so 30 whole Origin [cycles] have passed, and two Concordances out of the three in the current Origin. Thus we are at the start of the 3rd Concordance as stated.

[30] Ibid.

[31] Ibid., p. 1019. I.e. the first day of the Obscuration has the sexagenary name *jiazi*.1. Since this day is also an Era Head, we can deduce that the System Origin of this particular system likewise fell on a *jiazi*.1 day.

[32] Ibid.

a repetition of epochal conditions. So this is a consistent description of some Quarter [Day] system – but which one? Because

1624 – 1520 = 104

thus it is clear that 104 BC is an Era later than the Era Head in 1624 BC, and is therefore an Era Head itself, with coincidence of winter solstice and conjunction at midnight beginning a *jiazi*.1 day. These are precisely the conditions specified by the Quarter [day] system known to have been constructed by Sima Qian at that time – and taken over by the Grand Inception system, on which the Triple Concordance is based. So it seems that the Quarter [Day] system to which Liu Xin looks back is indeed the abortive creation of Sima Qian.[33] The fact however that the High Origin is so far back suggests that like the Triple Concordance this Quarter [day] system included provision for predicting planetary motions. It does not seem likely that Sima Qian himself did this – perhaps someone around the time of Liu Xin (or even Liu Xin himself) had at some stage experimented with a "planetary" version of Sima Qian's system, which would build on its predecessor as the Triple Concordance built on the Grand Inception system.

From the Zhou Conquest

Space forbids us from following Liu Xin's chronology of the Zhou conquest around 1122 BC in detail. But some of the most interesting and detailed retrodictions he makes are constructed on the basis of material he simply ascribes to "The Tradition 傳," but which we recognize today as coming from a passage in the *Conversations from the States Guoyu* 國語:

> Formerly when Wu Wang attacked Yin, the Year [star] was at Chunhuo, the moon was at Celestial Team, the sun was at the ford of Ximu, the conjunction was at the handle of Dipper, and the star [taken by all the commentators as the "water star" *shui xing* 水星, Mercury, in the light of the context] was at Celestial Tortoise. 昔武王伐殷, 歲在鶉火, 月在天駟, 日在析木之津, 辰在斗柄, 星在天黿.[34]

Let us follow the discussion of this material in the *Canon*.

> Wu Wang: the *Shujing, Mushi*, says that Wu Wang attacked Tchou of Shang. Water produces Wood, and so he acted in the virtue of Wood. His realm was named as the House of Zhou. In the Three Concordance system, from the High Origin to the year of the attack on Tchou, it is 142,109 years. Jupiter was at Chunhuo, in the

[33] Details are at *Shi ji* 史記 26, pp. 1260–61.

[34] *Guoyu*, *Zhou yu* 周語, B

> thirteenth *du* of Spread. 武王: 書經牧誓武王伐商紂. 水生木, 故爲木德. 天下號曰周室. 三統, 上元至伐紂之歲, 十四萬二千一百九歲, 歲在鶉火, 張十三度.[35]

The High Origin of the Triple Concordance is the winter solstice in late 143232 BC, preceding the Civil Year whose first month according to the usual civil (Xia) count was in spring 143231 BC. So the year in which the conquest occurred is:

(143232–142109) BC = 1122 BC

which is the figure already given above. The spreadsheet for the Triple Concordance confirms that at the winter solstice in late 1123 BC preceding this year Jupiter was in the position stated.

Shortly after this, we move into more complex calculations:

> When the army first set forth, it was in the eleventh Yin month, on day *wuzi*.25. The sun was in [the Jupiter station] Ximu, the 7th *du* of Winnower, and so the *Tradition* says "The sun was in Ximu." 師初發, 以殷十一月戊子, 日在析木箕七度, 故傳曰: "日在析木."[36]

Since the twelfth Shang/Yin month is that of the winter solstice, which is Xia month 11, then the eleventh Yin month is the tenth Xia month.

Now from the spreadsheet of the Triple Conjunction, in 1123 BC the tenth Xia month began on *xinyou*.58, and the conjunction fell at the third *du* of the lodge Heart . Thus day *wuzi*.25 fell 27 days later (since 58+27 = 60+25), and since the sun moves at 1 *du* per day, it would be at Heart 30, if Heart extended that far; since Heart in fact is 5 *du* wide and the next lodge, Tail, is 18 *du* wide, this places the Sun at Winnower 7 *du*, which is the twenty-fifth *du* of the station Ximu, all as required.

> On that evening, the Moon was in the 5th *du* of Chamber. Chamber is the Celestial Team. Thus the *Tradition* says " The Moon was in the Celestial Team." 是夕也, 月在房五度. 房爲天駟, 故傳曰: "月在天駟."[37]

In the Triple Conjunction system, the Moon moves

$254/19$ *du* per day = $20574/1539$ *du* per day;

so in the 27 days from the moment of conjunction to the same instant on *wuzi*.25 the Moon moves:

$27 \times 20574/1539 = 555498/1539$ *du*.

$= 360\ 1458/1539$ *du*

[35] *HS* 21A, 1015 [36] Ibid. [37] Ibid.

In the Triple Conjunction, a full circuit has

562120/1539 *du* =365 385/1539 *du*

So we are short of a full revolution by 4 466/1539 *du*.

Since the sun was at Heart 3 *du* at conjunction, it is clear that we are now 1 466/1539 *du* before the end of the preceding lodge, which is Chamber, which is five *du* wide. That would place us in the fourth *du*. Now the Lesser Surplus at the preceding conjunction was 67, which is the number of (1/81) of day from midnight to that instant, so we are already 67/81 through the day, which is about 20:00 local time. Since as already mentioned the moon moves 254/19 *du* per day, it is easy to calculate that the extra *du* we need will be moved within the next two hours, bringing us to the fifth *du* of Chamber before 22:00. So the moon is certainly at the required position "in the evening" of the day in question. Of course a moon 27 days old would not in fact rise until after midnight, so it would not have become visible at the moment in question. That is not apparently a relevant consideration here.

> Three days later one gets the Zhou first month, with conjunction at *xinmao*.28, with the mark-point for conjunction 1 *du* before Dipper, which is the Dipper Handle. Thus the Tradition says : "The mark-point was at the Dipper Handle." 後三日得周正月辛卯朔, 合辰在斗前一度, 斗柄也, 故傳曰: "辰在斗柄."[38]

The Triple Concordance system does indeed give the Zhou first month (Xia month 11) with conjunction at *xinmao*.28, at the tenth *du* of Winnower. Since Winnower is 11 *du* wide and Dipper comes next, this is indeed 1 *du* before Dipper as stated.

> The next day, *renchen*.29, Mercury first appeared. On *guisi*.30 Wu Wang first set out. On *bingwu*.43 he turned back his army, and on *wuwu*.55 he crossed Meng Ford. Meng Ford is 900 *li* from Zhou. The army traveled 30 *li* a day, so they crossed after 31 days. 明日壬辰, 晨星始見. 癸巳武王始發, 丙午還師, 戊午度于孟津. 孟津去周九百里, 師行三十里, 故三十一日而度.[39]

According to the Triple Conjunction spreadsheet, day *xinmao*.28 was 51,905,307 days from epoch. So the next day *renchen*.29 was 51,905,308 days from epoch.

The Triple Conjunction system assumes that at High Origin the two "inner" planets, Venus and Mercury, were just making their first morning appearance, meaning that they had moved sufficiently to the west of the sun to rise a little before it, and be visible for a short

[38] Ibid.

[39] Ibid.

time before the sun itself rose and blotted them out with its radiance. Thereafter, their return to this state was governed by the combination of two quantities:

Year Number: 9216 for Mercury
Appearance Number: 29041 for Mercury

The significance of these is that in 9,216 years, Mercury makes precisely 29,041 morning appearances. Since one year contains (562120/1539) days, the interval between morning appearances is:

$(9216/29041) \times (562120/1539)$ days

Each morning appearance is said to be followed sixty-five days later by the start of an evening appearance, when Mercury is to the east of the sun, and is visible setting in the west a little after sunset. If we look for the morning appearance preceding the date given here, we find that the days from High Origin will be, to the nearest whole day:

$447806 \times (9216/29041) \times (562120/1539)$ days = 51,905,243 days

Adding 65 days takes us to 51,905,308 days, which is indeed day *renchen*.29, the day specified above for an evening appearance of Mercury.

> On the next day, *jiwei*.56 winter solstice, Mercury set together with Woman. It had gone through Establishment Star [Dipper] and Ox, and had reached Woman, the head of the Celestial Tortoise. Thus the *Tradition* says: "The star was in Celestial Tortoise."
> 明日己未冬至, 晨星與婺女伏, 歷建星及牽牛, 至於婺女天黿之首, 故傳曰: "星在天黿."[40]

The Triple Concordance system gives winter solstice at the end of 1123 BC at day *jiwei*.56 as stated.

According to the Triple Concordance, from first morning appearance of Mercury to its evening setting is 65 + 26 = 91 days. Since the first appearance was on day *renchen*.29, then the disappearance should be close to day 29 + 91 − 60 = 60, four days later than winter solstice. Since at winter solstice the sun was at the first *du* of Ox, it would have been at the twenty-third *du* of Dipper four days before. Mercury is said to become invisible 15 *du* from the sun, so it must have then been at the thirty-eighth *du* of Dipper (extended), which is in fact the fourth *du* of Woman, which follows after the 8 *du* of Ox. Clearly at the winter solstice itself Mercury, still visible, could be said to be "setting with Woman."

[40] Ibid.

The movements ascribed to Mercury are correct, since on the day of the winter solstice the sun is at the start of Ox, and Mercury is to the east of it. Thus Mercury has passed through Ox. Establishment Star is equivalent to Dipper, which precedes Ox.

It seems extremely probable that the precise retrodiction of the events described here simply shows us that Liu Xin used what for him was a reliable dated record of Mercury's behavior at a distant period to adjust the values he had chosen for the Year Number and Appearance Number to give the best possible fit.

The Xinhai Year

The Zhou conquest was an obvious source of datable astronomical events for Liu Xin. Later, in the Spring and Autumn period, another event took place that was in itself an astronomical one, and it was a datum to which Liu Xin gave considerable importance as a reference point. This was the winter solstice of 656 BC, which preceded the regnal year that lay mostly in 655 BC. In the present *Zuo Zhuan* text the ruler of Lu is then named Xi 僖, but Liu Xin renders the name as 釐. The *Zuo Zhuan* itself describes how the duke ascended his "Numinous Tower" (Ling tai 靈臺) after announcing the first day of the month, in order to prognosticate from the visible pattern of vapours. The *Canon* simply notes the dates:

> In the fifth year of duke Xi, the first month, [on day] *xinhai*.48 there was conjunction and winter solstice. The Yin system made this *renzi*.49. It is 76 years from Duke Cheng. This year is distant from the High Origin by 142,577 years. One gets the 53rd Rule Head of the Meng Concordance. Thus the *Tradition* says "The fifth year, Spring, the Royal first month, *xinhai*.48, the sun was at its southern extreme." 釐公五年正月辛亥朔旦冬至, 殷曆以爲壬子, 距成公七十六歲. 是歲距上元十四萬二千五百七十七歲, 得孟統五十三章首. 故傳曰: "五年春王正月辛亥朔, 日南至."[41]

One immediate problem is with the years since High Origin. The count of regnal years given by Liu Xin establishes that this year is 655 BC, and we know that the first year of the Triple Concordance is 143231 BC. However:

143231 – 655 = 142,576

So the text of the *Canon* as we have it is in error so far as this figure is concerned. But at the solstice of 656 BC, preceding the stated

[41] Ibid., p. 1019. I.e. the sun was as low in the sky (for a northern hemisphere observer this means towards the south) as it ever gets – which means it was winter solstice.

regnal year 655 BC, conditions are as claimed in the text according to the Triple Concordance Since the distance from High Origin solstice to the 656 BC solstice in years is 142576 years, then we may cast out multiples of cycles as follows:

142,576 = 30x4617 + 2x1539 + 52x19

We see that we can cast out 30 Origins, 2 Concordances (so we are in the third Concordance of the current Origin), and exactly 52 Rules (so we are at the fifty-third Rule Head). Thus the 656 BC solstice is indeed the "fifty-third Rule Head of the Meng [3rd] Concordance," as stated. At a Rule Head, winter solstice and conjunction do fall on the same day, and the spreadsheet shows this as *xinhai*.48

Further, as stated, the Yin system gives coincidence of the 656 BC winter solstice and conjunction at midnight beginning day *renzi*.49 – which is an Obscuration Head. The text points ahead to the next Obscuration Head in seventy-six years time, under the reign of Duke Cheng, and when later Obscuration Heads of the Yin system are listed we are typically told the number of years that have elapsed "since the *xinhai*.48 [solstice]."

The purpose of this study is not to decide whether the conditions retrodicted by Liu Xin's calculations were accurate in the modern astronomical sense, but given the importance of this "*xinhai*" event for Liu Xin, it seems worthwhile to look at it for the moment. I give here data produced by the "Starry Night" desktop planetarium application (highly accurate for solar/lunar positions at this epoch), with local time at Chang'an taken to the nearest hour:[42]

> 656 BC winter solstice was at 21:00 on December 27 (*guichou*.50); preceding conjunction on 20:00 on December 26 (*renzi*.49).

This case is clearly one where contemporary calendrical prediction that winter solstice and conjunction both fell on *xinhai*.48 (December 25) would not have been obviously falsified. The view given by "Starry Night" for the morning of *xinhai*.48 shows only the faintest of crescents, with no more than 2 percent disc illumination, with the moon only 16 degrees from the sun and with the sun-moon line around 30 degrees to the horizon. Winter solstice error is much more difficult to detect, and at the period in question there would have been no reason not to be satisfied with the prediction for *xinhai*.48, since the sun's noon altitude is only two minutes more than the day of the solstice.

[42] The data were obtained by simply adjusting the time set until the required conditions could be seen to occur, and then reading the time from the on-screen display.

Some Solar Eclipses

By the beginning of the Eastern Han, it is clear that although solar eclipses were not expected to be predictable, records of solar eclipses were regarded as providing an important test for any astronomical system, since it was recognized that they could only occur at the conjunction of sun and moon. Here is one example of how Liu Xin dealt with such records:

> According to the *Spring and Autumn Annals*, Duke Cheng had been on the throne for 18 years, and then his son Duke Xiang, [named] Wu[43], came to the throne. The 27th year of Duke Xiang was distant from *xinhai* by 109 years. In the ninth month, day *yihai*.12 was the conjunction, and this is the month when *shen* is Established. The Clerk of Lu[44] wrote "12th month, *yihai*.12, conjunction, the sun was eclipsed"; the *Tradition* says "Winter, the eleventh month, *yihai*.12, conjunction, the sun was eclipsed, at which the mark-point was at *shen*.[45] Those in charge of the calendar went past the mark, and twice missed an intercalation." So that means that at the time the actual practice made this the eleventh month; they did not look into the Establishment, nor did they check it against the heavens." 春秋, 成公即位十八年, 子襄公午立. 襄公二十七年, 距辛亥百九歲. 九月乙亥朔, 是建申之月也. 魯史書: "十二月乙亥朔, 日有食之." 傳曰: "冬十一月乙亥朔, 日有食之, 於是辰在申, 司曆過也, 再失閏矣." 言時實行以爲十一月也, 不察其建, 不考之於天也.[46]

For Liu Xin, the problem is that the record of an eclipse on a *yihai*.12 day means that this day is guaranteed to have contained a real conjunction. So the Triple Concordance system has to retrodict it. But according to that system, the eleventh celestial month has conjunction on day *jiaxu*.11, which is one day earlier than the stated date of the eclipse, and the twelfth celestial month has conjunction on *guimao*.40, which is too far out to be explicable. So at first it would seem that the Triple Concordance fails.

But rather than admitting that his system might have been a day wrong in the eleventh month, Liu Xin prefers to look back to find a

[43] A duke of Lu is known to the annalists by a title conferred posthumously, which is not his personal name.

[44] Liu Xin would have been likely to have assumed that the officials in charge of the Lu calendar were more or less equivalent to the *shi guan* 史官 of his own day. The English term "clerk" has a combination of suggesting lowly literate functions while having more learned historical associations, and so I feel I can properly use it for *shi*.

[45] *Shen* 申 is the ninth of the twelve cyclical signs used to designate months; see above.

[46] Ibid., p. 1020.

month when the conjunction is retrodicted for a *yihai.*12 day – which he finds in the ninth Celestial month – and claim that two missed intercalations had led to this being counted as the eleventh month by the *Zuo Zhuan* (although he does not deal with the twelfth month record of the *Chunqiu* itself). If celestial month 1 is that of the "Establishment of Zi," then the ninth month corresponds to *shen* as stated.

Liu Xin's problem is to find a plausible month according to the Triple Concordance that fits the eclipse record in the text. For the modern reader, the problem is to find a day when an eclipse actually did occur in this year, and see if it matches the dates given. Now there was in fact no solar eclipse at the start of the Triple Concordance Ninth Celestial month, which was on August 14. However there was a visible solar eclipse in this year on 546 BC October 13, and this was a *yihai.*12 day – though the Triple Concordance makes this conjunction (that of the eleventh celestial month) fall on *jiaxu.*11. As James Legge was the first to point out,[47] it is far more likely that an error in an eclipse record would be in a month number than in the cyclical day.

One point that might have alerted Liu Xin that the eclipse actually occurred on the conjunction of the eleventh month as stated in the *Zuo Zhuan* is that the Triple Concordance predicted a lunar eclipse at the opposition preceding it. That would suggest that the conjunctions before or after might well have seen a solar eclipse, and since the tenth month is impossible, the eleventh month would have been indicated. But as already noted, Liu Xin felt that he had to insist on the correctness of the cyclical day his system gave for the conjunction.

This was an example of an eclipse record that Liu Xin felt able to accommodate within his system. In two other cases, however, he evidently did not feel able to deal with the situation, and passed by the records without mentioning them. The two *Zuo Zhuan* records are:

a. duke Shao, year 31 (511 BC): twelfth month, conjunction *xinhai.*48
b. duke Ding, year 5, (505 BC): third month, conjunction *xinhai.*48

For 511 BC the Triple Concordance system has the 12th month beginning on *gengxu.*47, not *xinhai.*48. The actual date of *xinhai.*48 would have been 511 BC, November 14, and the eclipse indeed occurred at around 09:15 local Chang'an time on November 14, 511 BC. The possibility of a solar eclipse at the conjunction of the twelfth month is signalled by the presence of a lunar eclipse prediction at the opposition of the eleventh month.

[47] James Legge, *The Chinese Classics* (rpt. Taipei: SMC Publishing, Inc., 1991), vol 5, pp. 86–87.

For 505 BC the Triple Concordance system has the third celestial month beginning on *jiyou.*46, not on *xinhai.*48. What is more, a two-month shift backwards to allow for possible missed intercalations does not find a *xinhai.*48 conjunction either. The eclipse (well visible from Chang'an), actually happened on Feb 16, 505 BC, which is on *xinhai.*48, as the record states. So the Lu calendar was hitting the right day for conjunction when the Triple Concordance system was two days out. Once again, the Triple Concordance predicts a lunar eclipse at the opposition preceding this conjunction.

In these cases, Liu Xin ignored eclipses he could not easily cope with. In one other case, he seems to refer to an eclipse that did not occur at all or at least that he ought not to have known *did* occur:

> Yuandi: The second year of the Chuyuan reign period, the eleventh month, [day] *guihai.*60 was conjunction and winter solstice. The Yin system made this *jiazi.*1, and made this an Era Head. But in this year, there was a solar eclipse in the 10th month, so this [winter solstice] was not a Coincidence with conjunction, and [thus] cannot be treated as an Era Head. 元帝初元二年十一月癸亥朔旦冬至, 殷曆以爲甲子, 以爲紀首. 是歲也, 十月日食, 非合辰之會, 不得爲紀首.[48]

As Liu Xin states, the Triple Concordance places the conjunction of the eleventh (Xia) month and winter solstice together on a *guihai.*60 day, which would be December 25, 48 BC, preceding the civil year that was the second of the Chuyuan period and which began in spring of 47 BC. The Yin system had them coinciding on the preceding day, *jiazi.*1 which was December 24. That would indeed be a Yin system Era Head.

It is not quite clear how the alleged eclipse in the tenth month was felt to damage this claim for the Yin system. No day is given, but it might be that the eclipse record Liu Xin saw placed it on the last day of the tenth month according to the Yin system, suggesting that the Yin system conjunction date was wrong. At any rate, no eclipse was visible from Chang'an near that date, nor is there any record of one in the *Han shu.* But it is striking that there was a real solar eclipse around that time, and it did occur on December 25. The only problem was that for an observer at Chang'an it was still well before dawn on that day when the eclipse occurred: to see it clearly, he would have had to be in a boat far out into the Pacific, well beyond the range of any coasting vessel that might have brought back a report. Something of interest

[48] *HS* 21A, p. 1024.

is clearly happening here, but without better details of the record to which Liu Xin refers so briefly it is hard to tell what it is. One interesting possibility is that Liu Xin retrodicted a solar eclipse for that date on the basis of an eclipse cycle, a procedure that can at best predict that a solar eclipse will be visible from *some* location without giving any assurance that it will be visible from any given location.[49]

THE BIRTHDAY OF THE OLD MAN OF JIANG COUNTY

Somehow it seems appropriate to end with this item from the *Canon*, in which Liu Xin discusses the problem of working out the year when a distinguished senior citizen was born:

> In the thirty-first year [of duke Xiang] the Year was at *jianglou* (station number 4). This year is 113 years from *xinhai*, and the second month had *guiwei*.20 [in it]. Above, it [*guiwei*.20] is distant from the year when in the eleventh year of duke Wen [the feudal lords] met at Chengkuang, the first Xia month *jiazi*.1 conjunction, by a total of 445 *jiazi* [cycles], and 20 odd days, which in days is 26,600 days and 6 ten-day cycles. Thus the *Tradition* says that the Old Man of Jiang county said "In the year that your servant was born, the conjunction of the first month was at *jiazi*.1, which is 445 *jiazi* [cycles]. From its time to now is one third [of a cycle] (that is, 20 days)." Music-master Kuang said: "That is the year when Yuchengzi had a meeting at Chengkuang, [which is now] 73 years [ago]." Clerk Zhao said "[The character] *hai* 亥 has two for its head and [a pair of] sixes for its body.[50] If you take down the two and put it with the body then that is his number of days." Shiwen Bo said "So that is 26,600 and 6 ten-day cycles." 三十一年歲在降婁. 是歲距辛亥百一十三年, 二月有癸未, 上距文公十一年會于承匡之歲夏正月甲子朔凡四百四十有五甲子, 奇二十日, 爲日二萬六千六百有六旬. 故傳曰絳縣老人曰: "臣生之歲, 正月甲子朔, 四百四十有五甲子矣. 其季於今, 三之一也." 師曠曰: "郤成子會于承匡之歲也,七十三年矣." 史趙曰: "亥有二首六身,下二如身,則其日數也." 士文伯曰: "然則二萬六千六百有六旬也."[51]

[49] I am indebted to Nathan Sivin for this suggestion (private communication, March 2003). For a detailed discussion of the possible use of eclipse cycles in this way, see Sivin, *Cosmos and Computation*, pp. 33–57. It would be a striking example of confidence in such cycles if Liu Xin had relied on such a retrodiction in the absence of any observational confirmation.

[50] In archaic forms of the character, the upper part of hai was two horizontal lines – hence "two." The lower part resembled a pair of counting-rod numerals for 6, each consisting of a horizontal rod with a vertical rod placed in contact with its mid-point, the upper with the vertical rod pointing up and the lower with the rod pointing down – as would be the case with two sixes in adjacent positions in a number. Thus it could be read as 266.

[51] *HS* 21A, p. 1021.

In year 31 of duke Xiang, 542 BC, Jupiter was indeed at *jianglou* according to the Triple Concordance system, and this was 113 years from 655 BC (the year of the *xinhai* winter solstice) as stated. The second Celestial month had conjunction on *yimao.*52 and thus contained day *guiwei.*20. Looking back to the meeting at Chengkuang, Year 11 of duke Wen was 616 BC; in that year the Triple Concordance system does indeed give the first Xia month with conjunction on a *jiazi.*1 day. So far all is clear and in order.

We may now turn to the two paragraphs on *jiazi* cycles. Now to get from the beginning of *jiazi* cycle number 1 to the start of cycle number 445 takes

444x60 days = 26,640 days,

and adding the 20 odd days gives us 26,660 days; this is indeed "26,600 days and 6 ten-day cycles" as stated. The problem is, however, that this does not appear to be the actual interval between the two dates named here.

Finding the interval between two days several years apart is best done using the astronomical system of Julian Day numbers (JD), which start at zero at Greenwich mean noon of January 1, 4713 BC. The Julian day number of midnight commencing the local day at Chang'an[52] on which the first Xia month conjunction, day *jiazi.*1, in 616 BC, falls is 1,496,470.20 (to the nearest hundredth).

Now since the third celestial month of 542 BC has conjunction on day *jiashen.*21, which commences locally at JD 1,523,490.20, therefore day *guiwei.*20 commenced at 1,523,489.20

1523489.20 – 1496470.20 = 27,019,

so the count of days including the initial *jiazi.*1 and the final *guiwei.*20 is 27020;

27,020 = 450x60 + 20

this is 450 cycles plus 20 days, which is six more cycles (360 days) than stated above.

Clearly Liu Xin's calculation has something wrong in it: it is as if an extra year has crept in. But where exactly? Let us consider another anomaly: the story of the Old Man says that the time interval is 73 years, when in fact from 616 BC to 542 BC is an interval of 74 years. That would certainly explain the extra year.

[52] I use Chang'an time when referring to Julian Days simply because my spreadsheet is set for Western Han conditions, when this was the capital. Of course the participants in this dialogue were not at Chang'an, but the fact that we are looking for a time interval in days rather than a time on any particular day renders this irrelevant.

Further, looking back to the original *Zuo Zhuan* entry (at least as we have it today) it places the old man's calculation as being made in the thirtieth year of duke Xiang, 543 BC, which is indeed 73 years from 616 BC, just as the Old Man said. So Liu Xin has shifted the year one forward by placing it in the thirty-first year. Moreover the *Zuo Zhuan* places the Old Man's statement on a *guiwei.*20 day in the third month rather than on the *guiwei.*20 day in the second month.

According to the Triple Concordance system, the third celestial month of 543 BC does not in fact contain *guiwei.*20, since it begins on *gengyin.*27. But on the simple assumption that the actual operative third month in the calendar in use in 543 BC was the Triple Concordance system's fourth month (easy, because of the differences of intercalation practice: the Triple Concordance system has intercalary month 12 in this year, but an actual intercalation one year earlier would produce this result), and since conjunction of the Triple Concordance system third month is on the *gengyin.*27 day beginning with midnight at JD 1523136.19750, we know that the *guiwei.*20 day (seven days earlier) in the second month began with midnight at JD 1523129.19750. So to get back to the *jiazi.*1 day on which the Old Man was born, we subtract 26,659 (allowing for counting start and finishing days), to obtain JD 1496470.19750 – which is our target day.

Looking for some explanation of this confusion, it seems likely that Liu Xin first did his year-by-year calculations of the Jupiter station, but then mistakenly copied in the old man's words after the thirty-first year, rather than the thirtieth year, and hence misled himself.

RETROSPECT

A sample of work in progress does not really deserve to be decorated with an elaborate list of conclusions. But there are a few thoughts that occur to me at this point in the unfinished explorations of which this essay forms a part.

Firstly, it is well to see Liu Xin's *Canon of the Ages* in the context of the project of which it was undoubtedly a part – the construction of a mathematized cosmos as a contribution to bolstering the legitimacy of his friend and patron Wang Mang. The basic solar and lunar parameters of the Triple Concordance were borrowed from the Grand Inception system, but the planetary constants produced by Liu Xin gave a new field for numerological virtuosity. Earlier in the *Han shu* we may read the summary proceedings, drafted by Liu Xin,[53] of the great confer-

[53] *HS* 21A, p. 955.

ence of cosmologists and experts in ancient texts summoned by Wang in AD 5, while he was still regent. There had already been a program of reconstruction of ancient ritual buildings under Liu Xin's supervision before the conference itself was summoned.[54] Given Wang Mang's desire to present himself as restoring the ancient virtue of the Zhou dynasty, what could be more impressive in astronomical terms than for Liu Xin to show that he had succeeded in recreating the system that underpinned the dates recorded in the most ancient texts recording the socially salvific acts of the heroes of the remote past, who had overthrown the cruelty of tyrants and brought order back to the world? Better still, that restored astronomical system of the ancient sages could be shown to embody the basic numerical structures of the cosmos: to decode the cosmos and decode history were one and the same thing.

That was surely one of the presentational imperatives behind Liu Xin's *Canon*. But few significant human activities serve a single unambiguous purpose. We must not ignore the possibility that he was also quite consciously testing his system on the most distant data he could find – which is not necessarily distinct from the complementary possibility that he had already used at least some of those data to set the parameters of his system. Liu Xin would thus have been using the ancient texts, as Ptolemy tells us that he used data provided by Hipparchus as "a groundwork of resources in the form of accurate observations from earlier times."[55] Liu Xin's was certainly not the first Chinese astronomer to do this (though no earlier examples have survived), but he is the earliest surviving example of what was to be a very long tradition.

Last of all, I offer one point in relation to the inclusion of an essay full of numbers in a collection such as this. The work of Liu Xin reminds us yet again that (as the Old Man suggests in the epigraph to this essay), any gentleman of education in ancient China might be expected to handle numbers freely, especially basic astronomical numbers. Innumeracy was strictly for *xiao ren*. We are unlikely to understand the totality of the world of Han if we exclude or marginalize aspects of it that were clearly central for some of its greatest thinkers. I hope this essay may have suggested that the effort needed to avoid this error need not be prohibitively great.

[54] *HS* 12, p. 359.

[55] *Almagest* IX.2 H210, in G. J. Toomer, trans., *Ptolemy's Almagest* (Princeton: Princeton U.P., 1998), p. 421.

LIST OF ABBREVIATIONS

HS *Han shu* 漢書